Performance in Addition and Subtraction Games with Decimal and Maya Numeration: An Experiment with Children of Different Ages

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Abstract:

This study explores how different numeration systems influence students' performance in addition and subtraction tasks. An experiment was conducted with primary and secondary school children using a web-based game with two versions: one using decimal notation and the other using Maya notation. Results show that children under 7 performed better with Maya notation, while older children showed increased performance with the decimal system. This suggests that students rely on the symbolic system rather than abstract grouping concepts. These findings highlight the importance of culturally diverse pedagogical strategies that foster conceptual understanding beyond a specific numeration system.

Keywords:

Numerical notation, ethnomathematics, decimal system, Maya numeration, mathematical education

Introduction

The teaching of mathematics has historically been viewed through a formal and abstract approach, focused on standardized and universal knowledge systems, whose goal is to transmit mathematical content and skills without considering the students' cultural context. However, in recent decades, various theories and approaches have emerged that aim to problematize and enrich mathematics education by recognizing the importance of local cultures, diverse forms of reasoning, and the relationship between mathematical knowledge and everyday life. Within this framework, the Theory of Objectification, semiotics in mathematics learning, and Guy Brousseau's Theory of Didactical Situations stand out, offering a dynamic vision of the process of mathematical knowledge construction.

The Theory of Objectification focuses on how students construct mathematical knowledge through interaction with mathematical objects and their symbolic representation. This approach highlights the importance of semiotics—understood as the study of signs and representational systems—in mathematics learning. Representations, such as numbers, operations, and diagrams, are tools through which students not only comprehend mathematics but also interpret and apply it in different contexts. Thus, mathematical knowledge becomes a construction process that involves both the individual and their environment, making mathematics learning an interactive and meaningful act.

On the other hand, the Didactics of Mathematics and the Didactical Situations proposed by Guy Brousseau promote an approach that places the student at the center of mathematical activity, considering problem-solving situations as the core of learning. Didactical situations are proposals that invite students to solve problems and, through this process, to construct and formalize their knowledge. This approach emphasizes the importance of providing students with a learning environment where they can interact with mathematical content and be active participants in the construction of their own knowledge.

Ethnomathematics, as defined by Ubiratan D'Ambrosio, is also part of this discussion, as it moves away from the idea of mathematics as a universal discipline and proposes that mathematics should be understood within specific cultural contexts. According to D’Ambrosio, ethnomathematics encompasses not only mathematical knowledge but also the knowledge systems, practices, and reasoning styles used by people from different cultures to solve problems and understand their environment. This approach highlights the importance of recognizing and valuing mathematics emerging from indigenous, rural, or minority cultures, which have been historically devalued or ignored in conventional education systems. In this sense, ethnomathematics offers a critical perspective on the universality of mathematics, showing that mathematical knowledge can be diverse, valid, and relevant to students.

Adopting ethnomathematics as a pedagogical approach invites students to learn mathematics through a cultural lens, understanding that mathematics is not merely an abstract tool but is closely linked to the realities and contexts of the communities that generate it. In this way, mathematics education becomes a tool for empowerment, allowing students to connect with their cultural identity and recognize that their own mathematical reasoning is just as valid as what is taught in academic institutions.

This approach, therefore, invites a deep critique of the universality of mathematics as a discipline and its teaching, opening the door to a more inclusive, diverse, and critical mathematics education—one that values mathematics from different cultural perspectives and promotes meaningful and empowering learning.

Methodology

Materials:

The study used a specially designed landing page to collect data on participants' ages and redirect them to the appropriate pages for the experimental activity. The page was designed to be accessible from both mobile devices and desktop computers, ensuring a simple user experience. The website was provided to a group of primary and secondary school students in the city of Morelia, Mexico.

The main page (landing page) included a form where users had to enter their age. Based on the age entered, the system redirected users to one of two versions of the activity:

- Page A: With decimal notation.
- Page B: With Maya notation.

The website was developed using HTML, CSS, and JavaScript, with a PHP backend to manage data registration. Additionally, a MySQL database was used to store responses and user progress.

Methods:

The experiment employed an experimental design in which participants were assigned to one of the two versions of the game sequentially. The objective was to explore whether the way numbers were represented (decimal on Page A and Maya on Page B) would influence students' performance in a simple addition and subtraction task.

Participants were primary and secondary school students, ranging in age from 5 to 14, which allowed the researchers to observe the impact of age on the comprehension and resolution of mathematical exercises using different numeration systems.

Procedures:

1. **Participant Selection:** The experiment was conducted with primary and secondary school students in the city of Morelia.

2. **Activity Start:** Upon accessing the website, participants filled out a form where they entered their age. This data was used to assign students to one of the two versions of the game. Assignment was done sequentially: if the visitor number was even, the participant was redirected to Page B (Maya numeration); if the number was odd, they were sent to Page A (decimal numeration).

3. **Game Development:**

- **Page A (Decimal Numeration):** Students on this page saw addition exercises using the decimal system (base 10).
- **Page B (Maya Numeration):** Students on this page solved the same type of exercises, but with numbers represented in Maya notation (without the digit zero).

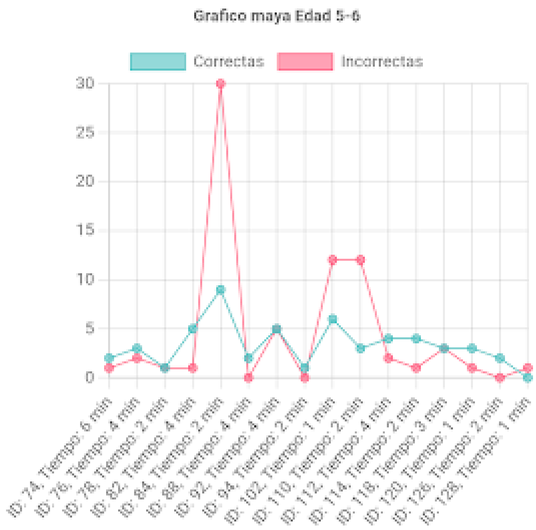
4. In both versions, participants received consecutive addition exercises. The goal of the study was not to measure how many sums they solved in a set time, but to observe how many exercises they chose to complete before exiting the game. Students could decide when to exit the game whenever they felt they had completed enough exercises.

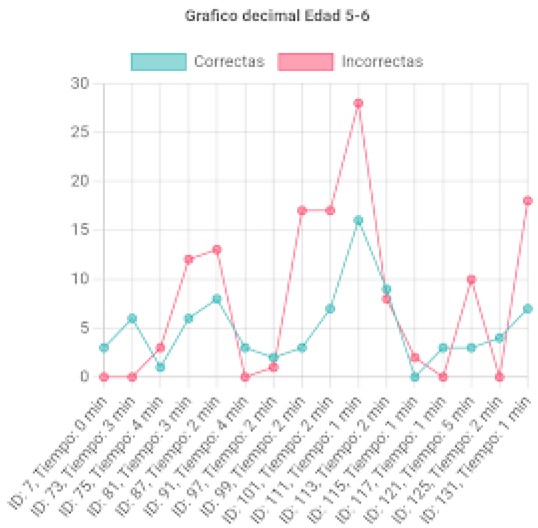
5. **Game Completion:** Upon exiting the game, a leaderboard appeared showing users' results, ranked by the number of correctly solved exercises. The leaderboard displayed the visitor numbers of users with the highest correct answer counts.

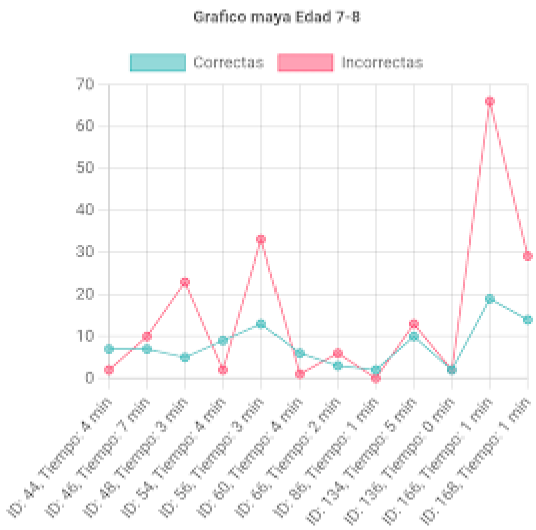
6. **Data Collection:** Data on the number of exercises participants chose to complete before closing the game, the number of correct and incorrect answers, and interaction time were stored in the database for later analysis.

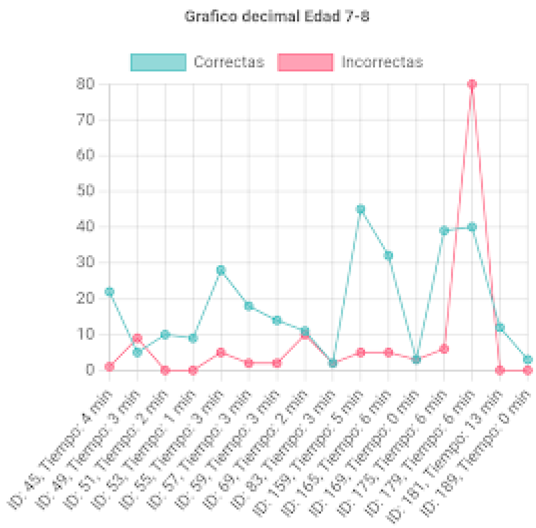
Results:

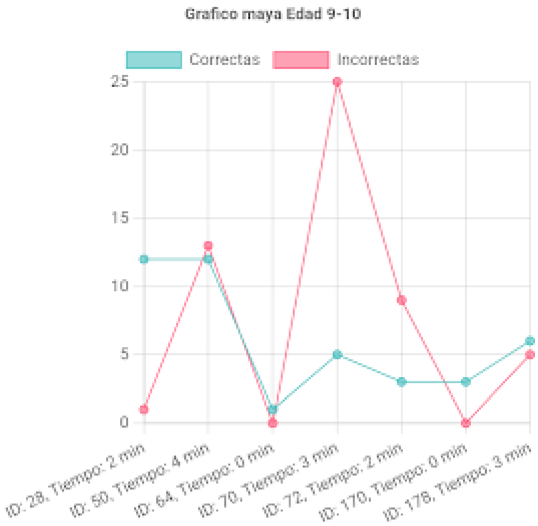
To analyze the observed patterns in participants' performance more precisely, the data was divided according to the students' age and the numeration system used (decimal or Maya). This categorization allows for the identification of significant differences in performance depending on the symbolic system and the children's cognitive development stage. Below are the corresponding graphs, which illustrate the average number of exercises solved correctly by participants in each group, as well as the general trends in interaction time and the number of exercises completed before closing the game. These visual representations facilitate the interpretation and comparison of the results obtained in both versions of the experiment.

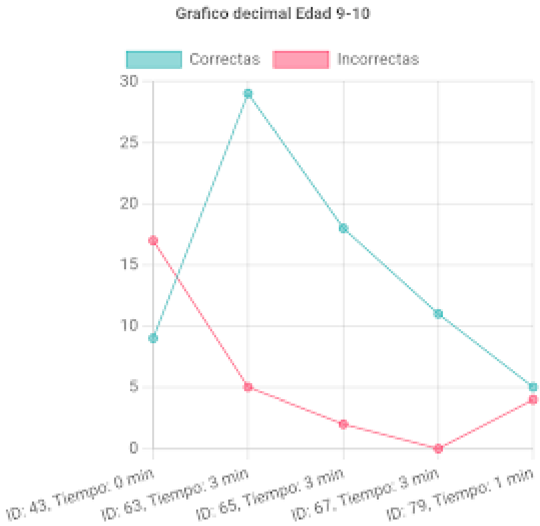


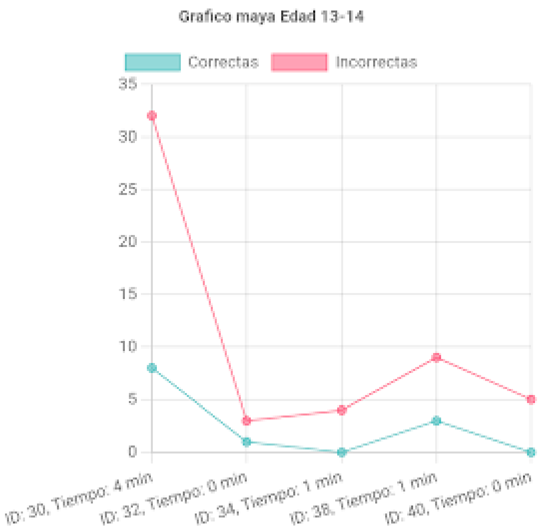












Discussion:

Interpretation and Analysis of Results

The results reflect interesting patterns that partially align with the study's objectives. In the graph corresponding to the 5–6-year-old age group using Maya notation, the curve of correct answers surpasses that of errors, indicating superior performance with this system for this age range. This finding is particularly relevant, as it suggests that younger children, whose thinking is still developing, may benefit from more visual and concrete symbolic systems, such as Maya notation.

In contrast, for all other age groups, this trend of outperforming errors with correct answers is only evident in the graphs corresponding to the decimal system. This indicates that as children grow older and gain greater familiarity with the decimal system in school, their performance in this system improves significantly. However, the fact that this improvement is not equally reflected in Maya notation suggests a lack of generalization of mathematical concepts, such as addition, across different symbolic systems.

Connection to Previous Research

These results align with Luis Radford's Theory of Objectification, which emphasizes the role of semiotic systems in the construction of mathematical knowledge. The data suggest that younger students may be more open to conceptualizing addition as the grouping of sets when presented with alternative visual representations, such as Maya notation. However, as older students specialize in the decimal system, they appear to rely more on this culturally dominant framework, consistent with Ubiratan D'Ambrosio's critiques of the perceived universality of Western mathematics.

Additionally, the results reinforce the importance of ethnomathematics, demonstrating that non-Western symbolic systems can facilitate learning at certain developmental stages. These findings underscore the need for a mathematical pedagogy that recognizes and values diverse cultural numeration systems.

Study Limitations

Small Sample Size in Secondary School: While the graphs for secondary school students show clear trends, the limited number of participants (n = 9) restricts the generalizability of results for these age groups.

Prior Exposure to Systems: Participants were more familiar with decimal notation due to its predominance in the educational system, which may have influenced the results.

Experiment Duration: The limited time to solve exercises may have impacted the number of completed responses, especially among younger children.

Geographic and Cultural Context: The results reflect the characteristics of a student group in Morelia, Mexico, and may not be representative of other cultural or educational contexts.

Implications for Future Research

Expanding the Sample: Conducting the study with a larger number of participants, particularly in secondary school age ranges, would yield more robust and representative data.

Exposure to Maya Notation: Investigating how prolonged prior exposure to Maya notation affects performance could explore whether mathematical knowledge generalizes better over time.

Multicultural Contexts: Replicating the study in contexts where alternative numeration systems (e.g., Maya notation) are more common could offer new insights into the relationship between culture and mathematical learning.

Longitudinal Analysis: A longitudinal study could track how students' ability to transfer mathematical concepts across numeration systems evolves over time.

Conclusion:

Key Findings:

- Children under 7 years old performed better with Maya notation, while older children excelled with the decimal system.
- Older students did not appear to generalize addition as the grouping of sets but rather relied on the symbolic system they were most familiar with.

Practical Recommendations:

- Design educational materials that incorporate culturally diverse numeration systems to promote abstract thinking and generalization skills.

Future Research Directions:

- Explore whether extended instruction in Maya notation could improve performance in older age groups.
- Investigate how other culturally relevant numeration systems might impact mathematical learning.

References:

Radford, L. (2003). "Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students’ types of generalization." Mathematical Thinking and Learning, 5(1), 37-70.

Radford, L. (2006). "Elements of a cultural theory of objectification." Revista Latinoamericana de Investigación en Matemática Educativa, 9(1), 103-129.

Brousseau, G. (1997). Theory of Didactical Situations in Mathematics. Kluwer Academic Publishers.

D'Ambrosio, U. (2004). Etnomatemática e interculturalidad: una reflexión sobre la enseñanza de las matemáticas en contextos de diversidad cultural. Revista Latinoamericana de Investigación en Matemática Educativa, 7(2), 113-130.

D'Ambrosio, U. (2001). Etnomatemática: A Intercultural Perspective on Mathematics Education. Nordic Studies in Mathematics Education, 7(1), 7-16.